



Creating a Space for Mathematicians and Educators: The philosophical basis for the Klein Project

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Resumo

O projeto de Klein é original dentro das iniciativas contemporâneas na instrução da matemática porque é projetado deliberadamente usar o potencial de ambos os matemáticos e educadores da matemática realçar a qualidade do ensino nos níveis superior da escola secundária. Isto significa que um "espaço" deve ser criado onde as discussões produtivas podem ser prendidas entre estes dois grupos, livres das divisões, dos pre-concepções, e da acrimónia que caracterizou algumas interações entre estes dois grupos. Igualmente significa que deve haver um mecanismo para o todo o aqueles interessado para fazer uma contribuição significativa. Eu discuto como este é conseguido.

Palavras chave: Projeto Klein, matemática como uma disciplina, educação continuada de professores.

Abstract

The Klein Project is unique within contemporary initiatives in mathematics education because it is deliberately designed to use the potential of both mathematicians and mathematics educators to enhance the quality of teaching at the upper levels of secondary school. This means that a "space" must be created where productive discussions can be held between these two groups, free from the divisions, pre-conceptions, and acrimony that has characterised some interactions between these two groups. It also means that there must be a mechanism for all those interested to make a significant contribution. I discuss how this is achieved.

Key words: The Klein Project, mathematics as a discipline, continuing education of teachers.

Introduction

The Klein Project was conceived from a separation, not from a joining together. The IMU General Assembly in 2006 agreed that the ICMI Executive should be elected by ICMI

member representatives, not by IMU member representatives, thereby effectively creating a wider separation between the communities of mathematics educators and mathematicians.

At the first meeting of the ICMI Executive after that decision in June 2007, members had a long discussion about this separation and its potential consequences. We agreed, first, that continued close relations between the two communities was essential for the health of both disciplines. We further agreed that these close relations could not be taken for granted, nor should such relations simply rest on the shoulders of those people who move easily in both camps. ICMI needed to actively engage in activities that brought together members of the two groups.

We brainstormed possible such projects. The Klein Project was one idea that met with universal approval. It also had emotional appeal: the 2008 centenary of ICMI was about to happen (Menghini, Furinghetti, Giacardi, & Arzarello; 2008), and Klein was ICMI's first President. Klein first published his famous work on mathematics education in that year (in German)(Klein, 1908), thus the project to revisit the idea of elementary mathematics from an advanced standpoint in the spirit of Klein was born. No, we did not envisage its effect, nor did we sit down and decide on a "philosophy" for the project, only its structure and some practicalities. (An extended description of the project can be found in (Barton, 2008) or on the website <<http://kleinproject.org>>).

Nevertheless, the project has already had some significant consequences, has developed its own character, and a philosophy has emerged. All bode well for continued productive relationship between the mathematics educators and mathematician communities.

Consequences

The immediate consequence was wide interest from a diverse group: teachers, educators, research and practicing mathematicians of all persuasions, amateur-mathematicians, graduate and undergraduate students. My personal view of this was not that the response was driven by a dissatisfaction with school mathematics (although such sentiments were expressed), but that mathematics, and its links to the school experience, is very close to the hearts of many, and can reach evangelical proportions. In its broad concept, the Klein Project has very wide appeal.

A developing consequence of the Klein Project is a need to struggle with the structure of contemporary mathematics. Tim Gower's Companion (2008) provides a research mathematician's view, but the idea of a structure for modern mathematics that incorporates its pedagogical development is a significantly different task. The Design Team's first attempt at this has echoed the structure of research mathematics, and the result does not feel completely right to me. An evolution with some input from a mathematics education perspective is necessary.

What do I mean a structure of mathematics incorporating pedagogical development? Allow me to give an example. Think about the *bête noir* of secondary mathematics classes: the quadratic equation. Why is this in the school curriculum? As a practical matter, it is probably there as an example of a polynomial function that can be handled analytically—we do not need advanced methods, either algebraic, geometric nor computational, that are

beyond the scope of a student of that age.

Quadratic equations per se are unlikely to be solved by most people in their future. But we do want all students to appreciate both the generality of mathematical structures and the explanatory power of elementary functions. The quadratic polynomial is an accessible example. However, quadratic equations (as a curriculum topic) is not just do-able algebra, nor is it a preliminary to cubic and higher order polynomials. It is also: an entrée to complex numbers (and thence quaternions), an example of a conic section, a potential introduction to the historical development of algebra, a powerful modeling tool, a student's first experience of the concept of a root, and so on.

Regarded mathematically only, quadratic equations may have all these links, but they are not part of their essential structure. How can the different perspectives on such topics in secondary level mathematics be captured in the advanced viewpoint?

A third consequence of the Klein Project has been the appropriation of the project by particular national or interest groups. For example, in Spain, the during Klein meeting in 2010, some participants invoked the spirit of the Klein Project in a separate discussion of recent national school curriculum changes. In New Zealand, a project is looking in a unified way at the four years that includes the final two years of secondary school and first two years of undergraduate mathematics. The "Envisioning" Project is also finding mechanisms to get teachers and research mathematicians together, and has adopting the idea of Klein vignettes in some of its writing. In Brazil, the Klein Project has been an organising idea around which significant government funding has been raised to mount workshops and seminars over a period of several years.

Such appropriation is warmly welcomed by the Klein Project. I regard the use of the concept by other groups as underlining the strength of the basic idea: that progress in mathematics education is much more likely when mathematicians, teachers, and mathematics educators get into bed together. We know also that the project itself will benefit directly by these activities.

Characteristics

The outstanding characteristic of the Klein Project is the positive energy of mathematicians and mathematics educators working alongside each other. They have put aside (sometimes significant) differences in opinions on curricular and assessment matters and contributed their mathematical thinking to the project. Strong debate has taken place, but neither destructive nor accusatory debate as has occurred in other forums.

The Design Team believes that the reason for the change is primarily the creation of a space in which members of each group have particular and acknowledged contributions to make. Research mathematicians do not know, in general, about the structure of school curricula, particularly its learning and teaching theory underpinnings. Mathematics educators do not know, in general, about the nature of research mathematics, particularly in the outer reaches of contemporary research. Both points of view are needed for the Klein Project, and the rights or wrongs of the current education system is not part of the issue at stake. A place has been created between secondary school learning and research mathematics in which everyone can participate.

Against the positive energy of having a project to work on together, there exists a frustration at the slow pace of progress. The project was conceived in 2007, formally begun in 2008, but over two years later has not produced very much actual writing. The causes are many, including the heavy calls on the time and energy of those leading it. However, despite a large amount of avowed interest, and a structure that allows anyone to contribute in written form at any time, very few contributions have been made, and only one or two such contributions are authored by people who are not in the Design Team.

Meetings have been held in many countries, and the list of those saying they will contribute is long. So why has it not happened? Part of the answer may be the collective philosophy of the writing. Rather than one or two designated authors, we are trying to produce a book/website that has collective and ongoing authorship. Are writers concerned that they will not be acknowledged? Are they concerned that they do not have the authority to write? Are they concerned to put their ideas about the wider field of mathematics in a public place when they are aware that such ideas are debatable? Or is it simply that, amid the myriad demands on teachers and university staff, an unpaid and un-rated research output simply does not compete?

Whatever the reason, the result is that the process (attending Klein Meetings) is much more successful than the product (producing Klein writing). An emerging thought is that perhaps the Klein Project's principle value will be these meetings and discussions, the creation of an awareness of the issues to be resolved in representing the field of mathematics through its educational phases, and the relationships developed between teachers and educators and mathematicians, including an appreciation of what each other actually does.

In terms of output, a significant feature of the Klein Project is the creation of vignettes. Conceived as a practical way of both getting the book/website written, vignettes have become both a defining characteristic of the project, and a basis for philosophical discussion. In attempting to create models of vignettes, the Design Team has been forced to face the essential nature of the project: to whom is it addressed, in what ways will it be useful, how can it be constructed so that it will be used, what are the criteria for deciding key ideas within the field, and how does contemporary mathematics link to its history and its applications? Such questions are far from being resolved.

Philosophy

The prime feature of the philosophy of the Klein Project is what has been termed the spirit of Felix Klein. By this is meant, on the one hand, the general conception of mathematics as a living, growing, interconnected organism, deeply related to its applications and the development of technology. The implication of this spirit is that mathematics education, and the educational knowledge of those teaching and pursuing mathematics, needs constant updating and renewal.

On the other hand, the spirit of Klein includes his faith in the important role of school teachers in development of the field. Klein trusted his teachers by explicitly saying that his book was not to be taken as a programme, or a guide. Rather it was to be a stimulus for them to draw from the field of mathematics inspiration for their teaching.

The process of the Klein Project also has philosophical underpinnings. The project is a

collective one—it has to be. No-one in the 21st century is in a position to understand the whole of the broader field of mathematics, all its directions, all its connections, all its history, all its big ideas. Even more, there is no-one with this broad mathematical sweep who also understands deeply the huge advances made in mathematical pedagogy in the last 100 years. We must work together if we are to even attempt a comprehensive account. Furthermore, since we are working on an assumption of an organic and linked field, it will not be sufficient simply to have many authors—they must be working in concert towards the same end. However difficult this may be, it is essential for success.

Finally, the product of the Klein Project also has some philosophical features. Apart from such characteristics as accessibility and universal appeal for its audience of mathematics teachers of 15-18 year old students worldwide, we seek to produce a resource that characterises our times: it must include its own mechanism for change, and it must use the resource technologies of our world. The current concept of both a printed book and a wiki-site (which might include a hyper-linked version of the book) captures both.

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